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D1-D5 black hole microstate counting from supergravity

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Abstract: We quantize the moduli space of regular D1-D5 microstates, directly from Type IIB SUGRA. The moduli space is parametrized by a smooth closed non-selfintersecting curve in four dimensions, and we derive that the components of the curve satisfy chiral boson commutation relations, with the correct value of the effective Planck constant previously conjectured using U-duality. We use the Crnković-Witten-Zuckerman covariant quantization method, previously used to quantize the 'bubbling AdS' geometries, combined with a certain new 'consistency condition' which allows us to reduce the computation to quantizing perturbations around the plane wave.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory.

Contents

1. Introduction

Black hole entropy can often be derived in string theory by counting excited states of equivalent D-brane configurations. The ensemble of D-brane states with fixed charges turns into a classical black hole when the Newton constant is increased. The main point of Mathur's 'fuzzball' idea [1] is to ask what happens with the individual (pure) D-brane states in the same process. AdS/CFT suggests that these may become smooth horizonless geometries. This opens up an exciting possibility: if these 'black hole microstate geometries' can be identified and counted, then the black hole entropy can be reproduced directly from supergravity. The goal of this paper is to do exactly this in the 2-charge black hole case, for which the microstate geometries are relatively well understood.

The plan of the paper is as follows. In section 2 we collect the necessary information about the D1-D5 black hole and its microstates, and formulate our main result. In section 3 we review the quantization method based on evaluating the symplectic form. In addition to our previous techniques [2, 3], we derive a 'consistency condition' which is a strong constraint on the form of the restricted symplectic form. In the present case, it can be used to predict the symplectic form up to a prefactor. To complete the calculation, it is sufficient to evaluate the prefactor for perturbations around a conveniently chosen specific spacetime, which we do in section 4. We conclude in section 5 with a general discussion of the future of Mathur's program.

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Note added. When this paper was being prepared for publication, we received an interesting paper [15] where, among other results, the 'planar curve' part of moduli space was quantized directly (using the method of [2]). Our result agrees with [15], while our new method based on the consistency condition makes the computation somewhat simpler.

2. D1-D5 geometries

2.1 Fields and charges

The D1-D5 black hole microstate geometries are regular solutions of Type IIB SUGRA. The nontrivial fields (the metric, the dilaton, and the RR two-form) are given by [4, 5]

$$\begin{aligned} ds^2 &= e^{-\Phi/2} ds^2_{\text{string}}\,, \\ ds^2_{\text{string}} &= \frac{1}{\sqrt{f_1 f_5}}\left[-(dt+A)^2+(dy+B)^2\right]+\sqrt{f_1 f_5}\,d\mathbf{x}^2+\sqrt{f_1/f_5}\,d\mathbf{z}^2\,, \\ e^{2\Phi} &= f_1/f_5\,, \\ C &= \frac{1}{f_1}(dt+A)\wedge(dy+B)+\mathcal{C}\,, \\ dB &= *_4 dA\,, \qquad d\mathcal{C} = -*_4\, df_5\,, \\ f_5 &= 1+\frac{Q_5}{L}\int_0^L \frac{ds}{|\mathbf{x}-\mathbf{F}(s)|^2}\,, \\ f_1 &= 1+\frac{Q_5}{L}\int_0^L \frac{|\mathbf{F}'(s)|^2 ds}{|\mathbf{x}-\mathbf{F}(s)|^2}\,, \\ A &= \frac{Q_5}{L}\int_0^L \frac{F_i(s)\,ds}{|\mathbf{x}-\mathbf{F}(s)|^2}\,dx^i\,. \end{aligned} \tag{2.1}$$

The solutions are asymptotically $M^5 \times S^1 \times T^4$; y and $\mathbf{z}$ denote the S^1 and T^4 directions. The moduli space is parametrized by a closed curve

$$x_i = F_i(s) \quad (0 < s < L, i = 1\dots 4), \tag{2.2}$$

which is assumed to be smooth and non-selfintersecting. Its parameter length has to satisfy

$$L = \frac{2\pi Q_5}{R}, \tag{2.3}$$

where R is the coordinate radius of S^1. Under these conditions the above geometries are completely regular. It should be noted that this description is somewhat redundant, since a constant shift of parameter $s \to s+h$ would produce the same geometry; this redundancy will play a role below.

In Mathur's approach to black hole microstates [1], solutions (2.1) are supposed to represent microstates of the spherically symmetric 2-charge geometry, which is given by eqs. (2.1) if we replace

$$f_{1,5} \to 1+\frac{Q_{1,5}}{|x|^2}, \quad A_i, B_i \to 0\,. \tag{2.4}$$

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This system is equivalent to $M^5 \times S^1 \times T^4$ with N_1 D1-branes wrapping S^1 and N_5 D5 branes wrapping the full $S^1 \times T^4$. The brane numbers are related to the charges by

$$Q_5 = g_s N_5, \quad Q_1 = \frac{g_s}{V_4} N_1 \tag{2.5}$$

(in the units $\alpha' = 1$), where g_s is the string coupling and $(2\pi)^4 V_4$ is the coordinate volume of T^4. The D1-D5 system is known to have the macroscopic entropy[1]

$$S = 2\pi\sqrt{2N_1 N_5}. \tag{2.6}$$

This entropy was not yet reproduced as a thermodynamic Bekenstein-Hawking entropy, because the spherically symmetric geometry has a classical horizon of zero area. A nonzero horizon is expected to appear once higher-curvature corrections to the two-derivative Type IIB SUGRA are included, similar to how it happens in the heterotic compactifications [7].

2.2 Counting D1-D5 microstates

In this paper we will reproduce (a finite fraction of) the entropy (2.6) by counting microstate geometries (2.1). To do this, we will have to quantize the curve $\mathbf{F}(s)$ (see footnote 3 in [5]). As we will show, in quantum theory $\mathbf{F}(s)$ acquires commutation relations:

$$\left[F_i(s), F'_j(\tilde{s})\right] = i\pi\mu^2 \delta_{ij}\delta(s - \tilde{s})\,. \tag{2.7}$$

This commutation relation has been previously conjectured by using the fact that the D1-D5 system can be U-dualized into the FP system (the multiply wound fundamental string carrying momentum along a compactified direction). The curve $\mathbf{F}(s)$ in the D1-D5 picture was identified with the profile of a chiral excitation of the dual string according to a simple proportionality relation

$$\mathbf{F}(s) = \mu \mathbf{F}_{\rm FP}(s), \qquad \mu = \frac{g_s}{R\sqrt{V_4}}\,. \tag{2.8}$$

The fundamental string is quantized using the quadratic action

$$\frac{1}{4\pi}\int (\dot{X}_i^2 - X_i'^2) d\tau d\sigma\,, \tag{2.9}$$

and thus satisfies the commutation relation

$$[X_i(\sigma), \dot{X}_i(\tilde{\sigma})] = i2\pi\delta(\sigma - \tilde{\sigma})\delta_{ij}\,. \tag{2.10}$$

The commutator of its chiral component is 1/2 of that:

$$[F_{{\rm FP}i}(s), F'_{{\rm FP}j}(\tilde{s})] = i\pi\delta_{ij}\delta(s - \tilde{s})\,, \tag{2.11}$$

and (2.7) follows.

Later in this paper we will show how, instead of such duality-based reasoning, one can derive (2.7) directly from the SUGRA action. At this point let us demonstrate how this

[1]See [6] for a derivation of this entropy by using the Dirac-Born-Infeld effective action to quantize supertubes.

result can be used to compute the degeneracy of the D1-D5 system. From the SUGRA point of view, we must count the number of microstate geometries with fixed charges $Q_{1,5}$, which translates into counting the number of curves $\mathbf{F}(s)$ satisfying the relation

$$Q_1 = \frac{Q_5}{L}\int_0^L |\mathbf{F}'(s)|^2 ds\,. \tag{2.12}$$

Classically, this question cannot be answered — there are infinitely many such curves. However, once we take the commutator (2.7) into account, we should expand $\mathbf{F}(s)$ into quantum oscillators:

$$\begin{aligned}
&\mathbf{F}(s) = \mu\sum_{k=1}^{\infty}\frac{1}{\sqrt{2k}}\left(\mathbf{c}_k e^{i\frac{2\pi k}{L}s} + \mathbf{c}_k^\dagger e^{-i\frac{2\pi k}{L}s}\right)\,,\\
&[c_k^i, c_{k'}^{j\dagger}] = \delta^{ij}\delta_{kk'}\,,\\
&\left\langle \int_0^L : |\mathbf{F}'(s)|^2 : ds \right\rangle = \frac{(2\pi)^2}{L}\mu^2 N_{\rm osc}\,,\\
&N_{\rm osc} = \sum_{k=1}^{\infty} k\langle \mathbf{c}_k^\dagger \mathbf{c}_k\rangle\,,
\end{aligned} \tag{2.13}$$

In such a quantum theory relation (2.12) takes the form

$$N_1 N_5 = N_{\rm osc}\,. \tag{2.14}$$

Thus the degeneracy of states is equal to the degeneracy of the N_1N_5 energy level in the system of 4 chiral bosons:

$$\Gamma \sim \exp\left(2\pi\sqrt{\frac{c}{6}N_1N_5}\right)\,, \quad c = 4\,. \tag{2.15}$$

It follows that the microstate geometries (2.1) account for a finite fraction of the full D1-D5 entropy. It is well known that they are insufficient to recover the full entropy: one would have to consider solutions corresponding to the string vibrating in the T^4 directions [5] and to the fermionic excitations of the string (see [8]).

3. Quantization from SUGRA

3.1 Symplectic form quantization

We would now like to derive the commutation relation (2.7) directly from SUGRA. There are several reasons why such a result would be welcome. First, it is important to know as a matter of principle that the commutators can be extracted from the classical geometries without any further input. Second, the U-duality-based derivation of (2.7) described above is not fully satisfactory: 1) it may not be easily generalizable to 3-charge microstate geometries which seem to have a very complicated moduli space parametrized by 4-dimensional hyper-Kaehler metrics [9]; 2) strictly speaking, the shape of the curve is not guaranteed to be a duality-invariant notion (only the degeneracies of states are).

Notice that we expect nontrivial commutation relations (2.7) to appear already *among the functions parametrizing the moduli space.* This situation should be contrasted with the case of Manton's soliton scattering, where positions of solitons by themselves commute, and to get nontrivial quantization one has to augment the phase space by adding momenta, corresponding to slow soliton motion. This difference can be traced to the stationary, as opposed to static, nature of our solutions.

The standard approach to quantization would be to work with the quadratic action for small fluctuations around every point in the moduli space. However, this is not technically feasible, since for a general geometry (2.1) there does not seem to be a convenient basis at hand into which to expand these small fluctuations. Another problem is that, typically, the quadratic action would couple fluctuations along moduli space to fluctuations orthogonal to it. This indicates that quantizing the quadratic action is a more difficult problem than the one we have to solve.

All these difficulties were explained in [2, 3], where it was proposed that instead of the quadratic action, it is more efficient to think directly in terms of the classical equivalent of the commutation relations — the Poisson brackets. In a general formulation of the problem we are given a classical dynamical system with phase space coordinates having the standard Poisson brackets

$$\{q^I, p^J\} = \delta^{IJ} . \tag{3.1}$$

We are also given a subspace $\mathcal{M}$ of the full phase space parametrized by some coordinates x^A:

$$q = q(x), \quad p = p(x) . \tag{3.2}$$

The problem is to find the induced Poisson brackets: $\{x^A, x^B\}$ =? To solve the problem let us consider the symplectic form of the theory:

$$\Omega = dp^I \wedge dq^I . \tag{3.3}$$

The symplectic structure on $\mathcal{M}$ is given by the pull-back (i.e. restriction) of Ω:

$$\begin{aligned} \Omega|_{\mathcal{M}} &= \omega_{AB}(x) dx^A \wedge dx^B , \\ \omega_{AB} &= \frac{\partial p^I}{\partial x^{[A}} \frac{\partial q^I}{\partial x^{B]}} . \end{aligned} \tag{3.4}$$

The induced Poisson brackets are simply given by the inverse of ω_{AB}:

$$\{x^A, x^B\} = \frac{1}{2}\omega^{AB} . \tag{3.5}$$

This argument shows why it is convenient to think in terms of the symplectic form: it encodes the Poisson brackets in a covariant way.

In our particular situation the phase space will be that of Type IIB SUGRA, the subspace $\mathcal{M}$ being the moduli space of D1-D5 geometries parametrized by closed curves. The Einstein-frame action of Type IIB SUGRA in the relevant sector is given by

$$S_{\rm IIB} = \frac{1}{(2\pi)^7 g_s^2} \int \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2} e^{\Phi} |F_3|^2 \right] , \quad F_3 = dC . \tag{3.6}$$

To define the phase space variables, we put the theory in the Hamiltonian form. Dynamical degrees of freedom on a surface of constant time will be given by the spatial components of the metric and of the two-form, as well as by the value of the dilaton:

$$q = \{g_{ab}, C_{ab}, \Phi\} \; . \tag{3.7}$$

The remaining components of the metric and of the two-form (g_{tt}, g_{ta}, C_{ta}) will appear in the action only as Lagrange multipliers, i.e. without time derivatives. The symplectic form of the theory is given by

$$\Omega = \int_{t=const} d^9x \sum_q \delta\Pi_q(x,t) \wedge \delta q(x,t), \quad \Pi_q = \frac{\partial L}{\partial \dot{q}} \, . \tag{3.8}$$

Here δ denotes the differential in the space of fields, not to be confused with the spacetime differential dx^μ. This equation can be rewritten in the Crnković-Witten-Zuckerman [10, 11] covariant formalism as an integral over a Cauchy surface Σ of the symplectic current:

$$\Omega = \int d\Sigma_\mu \, J^\mu, \tag{3.9}$$

$$J^\mu = \delta\left(\frac{\partial L}{\partial \partial_\mu \psi_A}\right) \wedge \delta\psi_A \, . \tag{3.10}$$

Here ψ_A runs over all the fields of the theory. If we choose $\Sigma = \{t = const\}$, the contributions of Lagrange multipliers drop out, and we recover (3.8). The symplectic current J^μ has several useful properties: 1) it is conserved as a consequence of the classical equations of motion (and thus Ω is independent of the choice of the Cauchy surface), it changes by a total derivative under a gauge transformation (and thus Ω is gauge invariant). Moreover, both J^μ and Ω are invariant under point transformations of the elementary fields.

The symplectic form of the relevant sector of Type IIB SUGRA can be written as

$$\Omega = \frac{1}{(2\pi)^7 g_s^2} \int d\Sigma_\mu \, J^\mu \, ,$$

$$J^\mu = J^\mu_g + J^\mu_F + J^\mu_\Phi \, . \tag{3.11}$$

The three terms here are the gravity, two-form, and dilaton symplectic currents, which can be computed using eq. (3.10) from the corresponding parts of the action (3.6). We have

$$J^\mu_g = -\delta\Gamma^\mu_{\alpha\beta} \wedge \delta(\sqrt{-g}g^{\alpha\beta}) + \delta\Gamma^\beta_{\alpha\beta} \wedge \delta(\sqrt{-g}g^{\mu\alpha}) \, , \tag{3.12}$$

$$J^\mu_F = -\delta\left(\sqrt{-g}e^{-\Phi}F^{\mu|\alpha\beta|}\right) \wedge \delta C_{|\alpha\beta|} \, , \tag{3.13}$$

$$J^\mu_\Phi = -\delta\left(\sqrt{-g}\partial^\mu\Phi\right) \wedge \delta\Phi \, . \tag{3.14}$$

The J^μ_g is known as the Crnković-Witten current [10]. When deriving it from the gravitational action, one should use the so-called $\Gamma\Gamma - \Gamma\Gamma$ Lagrangian, which contains only the first derivatives of the metric and differs from the standard Einstein-Hilbert Lagrangian by a total derivative term.

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3.2 Consistency condition

The above logic has been laid out in [2, 3], where it was used to quantize the moduli space of the 'bubbling AdS' geometries [12]. It can be used to compute the symplectic form on any subspace of the full phase space of gravity. Now we will add a new ingredient to the discussion. Namely, we observe that the subspace we are dealing with is not just *any* subspace: it is rather special in that it consists of time-independent solutions. It turns out that in this case there is an additional piece of information which can simplify the derivation of the restricted symplectic form. In particular, this information will allow us to predict the resulting symplectic form up to a coefficient, without doing any difficult computations.

In fact, the idea we are about to explain applies to any subspace $\mathcal{M}$ which is invariant under the Hamiltonian evolution. Thus we consider a general Hamiltonian system (H, Ω) with a Hamiltonian H and a symplectic form Ω. Let us restrict Ω and H to $\mathcal{M}$:

$$\omega = \Omega|_{\mathcal{M}}, \qquad h = H|_{\mathcal{M}}\,. \tag{3.15}$$

Now we have two hamiltonian flows on $\mathcal{M}$: the original flow (H, Ω), which leaves $\mathcal{M}$ invariant by assumption, as well as the flow (h, ω) generated by the restricted objects.

Theorem (*Consistency condition*): These flows are equivalent on $\mathcal{M}$: $(H,\Omega) \equiv (h, \omega)$.

The proof is very simple. The solutions of the Hamilton equations can be obtained in the first-order formalism as stationary curves of the functional

$$\int dt \left[K_i(X)\dot{X}^i - H(X)\right] , \tag{3.16}$$

where K is a one-form such that $\Omega = dK$. Restricting Ω to $\mathcal{M}$ is equivalent to restricting K. A stationarity curve will of course remain stationary under a smaller class of variations which do not take the curve outside $\mathcal{M}$. It follows that any curve of the flow (H, Ω) must satisfy equations of motion of (h, ω). Q.E.D.

To apply this theorem in practice, we must evaluate the on-shell Hamiltonian (i.e. the solution energy) $H|_{\mathcal{M}}$ as a functional on the moduli space. From the form of this functional we get a *consistency condition* which should be satisfied by ω: the Hamiltonian equations derived from $H|_{\mathcal{M}}$ and ω should coincide with the evolution on the moduli space implied by the form of the solutions.

The energy of the D1-D5 microstate geometries is given by

$$H|_{\mathcal{M}} = \frac{RV_4}{g_s^2}\left(\frac{Q_5}{L}\int_0^L |\mathbf{F}'(s)|^2 ds + Q_5\right) . \tag{3.17}$$

It can be evaluated using the standard general relativity formula for the asymptotically flat spacetimes

$$H = \frac{1}{(2\pi)^7 g_s^2}\int_{\partial\Sigma}\left(\frac{\partial g_{ab}}{\partial x^b} - \frac{\partial g_{bb}}{\partial x^a}\right) n^a \tag{3.18}$$

where $\partial\Sigma$ is the asymptotic boundary, which in our case is $S^1 \times T^4$ times the 3-sphere at large $|x|$, g_{ab} is the spatial metric (i.e. a, b run over the coordinates $\mathbf{x}, y, \mathbf{z}$), and n^a is the outside unit normal to Σ. Notice that (3.17) agrees with the total mass of the D-branes:

$$E_{\rm tot} = g_s^{-1}(N_1 R + N_5 R V_4)\,.$$

The consistency condition implies that the Poisson brackets on the moduli space should be such that the Hamilton equation

$$\frac{dF_i}{dt} = \{F_i, H|_{\mathcal{M}}\} \tag{3.19}$$

be compatible with the time-independence of the microstate geometries. A moment's thought shows that the only nontrivial allowed equation is

$$\frac{dF_i}{dt} = \text{const.}\frac{dF_i}{ds}\,, \tag{3.20}$$

so that

$$F_i(s,t) = F_i(s + \text{const.}t)\,. \tag{3.21}$$

Such, and only such, dynamics leads to time-independent geometries, since the metric and other fields are given in terms of contour integrals which remain unchanged under the constant shift of parameter (3.21).

The Hamiltonian (3.17) leads to eq. (3.20) if and only if the Poisson brackets have the form:

$$\{F_i(s), F_j'(\tilde{s})\} = \alpha\,\delta_{ij}\delta(s-\tilde{s})\,, \tag{3.22}$$

where α is a constant. The corresponding symplectic form is thus fixed up to a proportionality coefficient[2]:

$$\Omega = \frac{1}{2\alpha}\int \delta F_i'(s) \wedge \delta F_i(s)\, ds\,. \tag{3.23}$$

Notice that the precise value of α cannot be established by this argument — it will have to be found by an explicit calculation. Notice also that α, although a constant for each $\mathbf{F}(s)$, could in principle depend on $\mathbf{F}(s)$ by being a function of integrals of motion (e.g. of the Hamiltonian or higher-derivative contour integrals):

$$\alpha = \alpha\left(\int \mathbf{F}'^2 ds, \int \mathbf{F}''^2 ds, \int K(s-\tilde{s})|\mathbf{F}(s) - \mathbf{F}(\tilde{s})|^2 ds d\tilde{s}, \dots\right)\,. \tag{3.24}$$

The only requirement is that it should be invariant under the shifts of parameter $\mathbf{F}(s) \to \mathbf{F}(s+h)$. Thus the calculation also has to establish that α is a numerical constant, and more precisely that $\alpha = \pi\mu^2$. In this case the Poisson bracket (3.22) promotes upon quantization to precisely the conjectured commutator (2.7).

[2]Notice that this symplectic form is invariant under the infinitesimal transformation $\delta\mathbf{F}(s) \to \delta\mathbf{F}(s) + \epsilon\mathbf{F}'(s)$. Thus it restricts nicely to the 'true' moduli space which is the space of curves modulo constant parameter shifts. In fact the whole above discussion could be phrased, completely equivalently, in terms of this 'true' moduli space.

To conclude this section, we would like to note that in the 'bubbling AdS' case mentioned above, where the symplectic form was computed using the direct CWZ method in [2, 3], the consistency condition turns out to be much more powerful and in fact fixes the symplectic form completely, i.e. together with the prefactor. The reason is that in the 'bubbling AdS' case there is non-trivial dynamics on the moduli space of geometries, namely the planar droplets parametrizing it rotate with a particular angular velocity, which can be fixed unambiguously by requiring that the metric take the asymptotically AdS form with all perturbations going to zero at the prescribed rate[3].

4. Fixing the prefactor

4.1 Simplifying assumptions

In the previous section we used the consistency condition to show that the time-independence of the D1-D5 solutions together with the form that the Hamiltonian takes on the moduli space leave very little freedom for the restricted symplectic form: it *must* be given by eq. (3.23), and the only thing that remains is to evaluate the coefficient α. This is quite a strong restriction, since the most general expression for the symplectic form respecting the translation invariance in the parameter space is

$$\Omega_{\text{general}} = \int ds\, d\tilde{s}\, K_{ij}(s-\tilde{s}|\mathbf{F})\, \delta F_i(s) \wedge \delta F_j(\tilde{s})\,, \tag{4.1}$$

where the *symplectic kernel* K has to be antisymmetric:

$$K_{ij}(s|\mathbf{F}) = -K_{ji}(-s|\mathbf{F})\,. \tag{4.2}$$

Some extra restrictions could be generally derived using the invariance of Ω under the Poincaré group acting on the curve, but these restrictions would fall far short of what we have established: that the kernel is diagonal in the perturbations and, most importantly, that it is *local*:

$$K_{ij} = \frac{1}{2\alpha(\mathbf{F})}\delta_{ij}\delta'(s)\,. \tag{4.3}$$

Although an explicit calculation will be necessary to determine α, it turns out that what we already know will allow us to organize this calculation in a rather economical way.

Rather than evaluating the symplectic form in full generality, as it was done in [2] for the 'bubbling AdS' case, the idea is to find a case which is simple yet restrictive enough so that it fixes the remaining freedom. We will consider the following set of simplifying assumptions : 1) the curve contains a straight-line interval $\mathcal{I}$ of the form (see figure 1):

$$F_1(s) = s, \quad F_{2,3,4}(s) = 0 \quad (0 < s < 1)\,; \tag{4.4}$$

2) the only nonzero component is

$$\delta F_2(s) \equiv a(s) \quad (0 < s < 1)\,. \tag{4.5}$$

[3]The question whether it is possible to make such an argument was first posed to me by Boris Pioline.

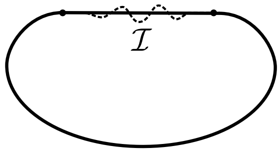

Figure 1: The class of closed curves containing a unit interval $\mathcal{I}$ along x_1, and otherwise having an arbitrary profile. We evaluate the symplectic form around any curve from this class, and for the perturbations supported on $\mathcal{I}$ and directed along x_2. This symplectic form is then extended to the full moduli space by uniqueness.

Under these assumptions, we will show that the symplectic form is given by

$$\Omega = \frac{1}{2\pi\mu^2} \int a'(s) \wedge a(s)\, ds\,. \tag{4.6}$$

independently of the shape that the curve takes outside $\mathcal{I}$. A moment's thought shows that this is enough to rule out any nontrivial dependence of the kind (3.24). In other words, the only possible expression of the form (3.23),(3.24) which reduces to (4.6) under the above simplifying assumptions is the one where $\alpha = \pi\mu^2$ is a numerical constant independent of $\mathbf{F}(s)$.

4.2 Field perturbations

Having assumed (4.4),(4.5), let us begin the evaluation of the symplectic form. First of all we have to compute perturbations of the fields $f_{1,5}, A, B, \mathcal{C}$ in (2.1). It will be convenient to introduce the polar coordinates in the space transverse to $\mathcal{I}$:

$$\begin{aligned} dx_i^2 &= dx_1^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \\ r &\equiv \sqrt{x_2^2 + x_3^2 + x_4^2}\,, \quad x_2 = r\cos\theta\,. \end{aligned} \tag{4.7}$$

Starting with f_5, we can write its perturbed value as follows:

$$\begin{aligned} f_5(\mathbf{F} + \delta\mathbf{F}) &= \bar{f}_5 + \frac{Q_5}{L} \int_{-\infty}^{\infty} \frac{ds}{(x_1 - s)^2 + (x_2 - a(s))^2 + x_3^2 + x_4^2} \\ &\approx \bar{f}_5 + \frac{\pi Q_5}{Lr} + \frac{Q_5}{L} \int_{-\infty}^{\infty} \frac{2x_2 a(s)\, ds}{[(x_1 - s)^2 + r^2]^2}\,, \end{aligned} \tag{4.8}$$

$$\bar{f}_5 \equiv 1 + \frac{Q_5}{L} \int_{s\notin\mathcal{I}} ds \left(\frac{1}{|\mathbf{x} - \mathbf{F}(s)|^2} - \frac{1}{(x_1 - s)^2 + r^2} \right)\,, \tag{4.9}$$

where $\approx$ means that we expanded to the first order in $a(s)$. The $\bar{f}_5$ is the regular part of f_5 (i.e. it has no singularities on $\mathcal{I}$); it does not vary with $a(s)$. Using the momentum representation for the convolution integral, we have

$$\begin{aligned} f_5 &\approx \bar{f}_5 + \gamma f^{\mathrm{sing}}, \\ f^{\mathrm{sing}} &\equiv \frac{1}{r} + \frac{\cos\theta}{r^2} \left[(1 + r|p|)\, e^{-r|p|} \tilde{a}(p) \right]^{\vee}\,, \end{aligned} \tag{4.10}$$

$$\gamma \equiv \frac{\pi Q_5}{L} = \frac{R}{2}. \tag{4.11}$$

Here we introduced the notation for the Fourier transform in x_1 and its inverse:

$$\tilde{a}(p) \equiv \int ds\, e^{ips} a(s), \qquad b(x) = \left[\tilde{b}(p)\right]^\vee \equiv \int \frac{dp}{2\pi}\, e^{-ipx_1}\tilde{b}(p)\,.$$

Analogously we get

$$f_1 \approx \bar{f}_1 + \gamma f^{\mathrm{sing}}\,, \tag{4.12}$$

$$A \approx \bar{A} + \gamma f^{\mathrm{sing}} dx^1 - \gamma \frac{\cos\theta}{r}\left[ipe^{-r|p|}\tilde{a}(p)\right]^\vee dx^2\,, \tag{4.13}$$

where the barred fields, whose precise form will not be needed below, are again regular on $\mathcal{I}$. The B and $\mathcal{C}$ are then found by solving the flat-space Hodge dual equations in (2.1):

$$B \approx \bar{B} + \gamma\left\{1 - \cos\theta + \frac{\sin^2\theta}{r}\left[(1 + r|p|)\, e^{-r|p|}\tilde{a}(p)\right]^\vee\right\} d\phi\,, \tag{4.14}$$

$$\begin{aligned}\mathcal{C} \approx\ & \bar{\mathcal{C}} + \gamma\left\{-1 + \cos\theta - \frac{\sin^2\theta}{r}\left[e^{-r|p|}\tilde{a}(p)\right]^\vee\right\} dx^1 \wedge d\phi + \\ & + \gamma\cos\theta\sin\theta\left[(i\,\mathrm{sign}\, p)\, e^{-r|p|}\,(2 + r|p|)\,\tilde{a}(p)\right]^\vee d\theta \wedge d\phi\,. \end{aligned} \tag{4.15}$$

4.3 A coordinate transformation

If we substitute (4.10)–(4.15) into (2.1) and expand in $a(s)$, we will find what we call 'naive' perturbations of $g_{\mu\nu}, C_{\mu\nu}, \Phi$. Schematically we have:

$$\delta g^{\mathrm{naive}}_{\mu\nu} = g_{\mu\nu}(\mathbf{F} + \delta\mathbf{F}) - g_{\mu\nu}(\mathbf{F}) \tag{4.16}$$

expanded to the first order in $\delta\mathbf{F}$, with analogous expressions for Φ and C. These perturbations will not yet be suitable for computing the symplectic form using eq. (3.11). The reason is that $\delta g_{\mu\nu}$ so defined will be singular on the curve in the coordinate system in which $g_{\mu\nu}$ is regular. A manifestation of this singularity is that the $O(a(s))$ terms in (4.10)–(4.15) are all more singular as $r \to 0$ than the zero-order terms. The correct definition for the field perturbations is ([2])

$$\delta g_{\mu\nu} = g^{(\varepsilon)}_{\mu\nu}(\mathbf{F} + \delta\mathbf{F}) - g_{\mu\nu}(\mathbf{F})\,, \tag{4.17}$$

where an appropriate change of coordinates $x^\mu \to x^\mu - \varepsilon^\mu$ has to be applied to $g_{\mu\nu}(\mathbf{F} + \delta\mathbf{F})$ before the subtraction is made. The effect of this can be expressed as

$$\delta g_{\mu\nu} = \delta g^{\mathrm{naive}}_{\mu\nu} + \varepsilon^\lambda g_{\mu\nu,\lambda} + 2\varepsilon^\lambda_{,(\mu} g_{\nu)\lambda}\,, \tag{4.18}$$

where the additional terms (which can also be written as $\nabla_{(\mu}\varepsilon_{\nu)}$) are the effect of an $O(a)$ coordinate transformation. They have to be chosen so that the resulting $\delta g_{\mu\nu}$ be regular. Analogously we will have

$$\delta\Phi = \delta\Phi^{\mathrm{naive}} + \varepsilon^\lambda \Phi_{,\lambda}\,, \tag{4.19}$$

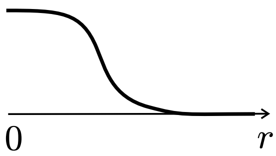

Figure 2: The 'cutoff' C^∞ function $\chi(r)$ interpolates smoothly between 1 at $r = 0$ and 0 at $r = \infty$.

$$\delta C_{\mu\nu} = \delta C^{\rm naive}_{\mu\nu} + \varepsilon^\lambda C_{\mu\nu,\lambda} + 2\varepsilon^\lambda_{,(\mu} C_{\nu)\lambda} + \Lambda_{[\mu,\nu]}\,, \tag{4.20}$$

where in the case of the two-form field we also have to include the effect of an abelian gauge transformation.

It is not difficult to guess the form of a coordinate transformation $x \to \tilde{x}$ needed to make the field perturbations regular. The effect of this transformation should be such that 1) the perturbed curve $x_2 = a(x_1)$ has equation $\tilde{x}_2 = 0$ in the new coordinates; 2) the transformation has unit Jacobian on the curve; 3) the transformation tends to the identity transformation at infinity. The following transformation satisfies all these requirements and will do the job:

$$\begin{aligned} \tilde{x}_1 &= x_1 + a'(x_1) x_2 \chi(r)\,, \\ \tilde{x}_2 &= x_2 - a(x_1)\chi(r)\,. \end{aligned} \tag{4.21}$$

Here $\chi(r)$ is any function which interpolates smoothly between 1 at $r = 0$ and 0 at $r = \infty$ (see figure 2). Thus the only nonzero components of ε^μ are

$$\varepsilon^1 = -a'(x_1) x_2 \chi(r), \quad \varepsilon^2 = a(x_1)\chi(r)\,. \tag{4.22}$$

4.4 Reduction to the plane wave

Implementing this coordinate transformation, we will get regular field variations, which could in principle be used to compute the symplectic form using eq (3.11). Note that the barred terms in (4.10)–(4.15) will 'contaminate' the field variations as a consequence of expansions that will have to be done once (4.10)–(4.15) is substituted into (2.1), and later via the second terms in (4.18)–(4.20). However, we claim that all these contaminating contributions can be ignored. More precisely, we would like to argue that the symplectic form can be computed by putting all the barred terms to zero both in the unperturbed spacetime and in the perturbations.

The argument why this is so consists of three steps. First of all, we notice that to determine the *local* contribution to the symplectic current, it is enough to perform the integration in (3.9) in a small neighborhood of $\mathcal{I}$, which should include the regions where $\chi \neq 0$. The reason is that outside of this neighborhood the field perturbations depend on $a(s)$ in a *regular nonlocal* way. Thus the same will be true of the symplectic current, and upon integration this region can only give a *nonlocal* contribution to the symplectic form. Such a contribution cannot affect the coefficient in front of the *local* symplectic kernel (4.3), which we have to determine.

Second, once we have limited ourselves to a small neighborhood of $\mathcal{I}$, in this region we can expand the barred fields in the transverse space. Schematically we can write:

$$\bar{f} = \bar{f}|_{r=0} + x_\perp^i \partial_i \bar{f}|_{r=0} + \cdots , \qquad x_\perp^i = (x_2, x_3, x_4) , \tag{4.23}$$

where $\bar{f}$ stands for any of the barred fields that we encountered. The coefficients in this series are fields of $x_1 = s$ only. Now, it is easy to check that the metric and the perturbations will depend on these coefficients analytically near $\mathcal{I}$. In particular, the limit when all the barred fields tend to zero is completely well behaved. Away from this limit, the coefficients of (4.23), say $\bar{f}(s)$, will enter the field perturbations as functions multiplying $a(s)$, $a'(s)$, or general convolution integrals of the form

$$\int a(s-\tilde{s}) \Psi\left(\frac{\tilde{s}}{r}\right) d\tilde{s} , \tag{4.24}$$

evaluated *at the same point* s. It means that after integration, these coefficients may make their way into the symplectic form only via terms involving products $\bar{f}(s)a(s)$ and such.

As a third, final, step, we notice that there is a lot of freedom to deform the curve outside $\mathcal{I}$ to make the barred fields into functions varying nontrivially along and near $\mathcal{I}$. This can be seen by examining the explicit expression (4.9) for $\bar{f}_5$ and analogous expressions for the other fields. However, for varying $\bar{f}$, the terms involving products like $\bar{f}(s)a(s)$ would not be consistent with the translationally invariant nature of the symplectic kernel (4.3). This means that such terms simply cannot appear, which finishes the argument.

Putting the barred fields to zero means that we pass from the closed curve to an infinite straight line and also drop the 1's from $f_{1,5}$. The resulting nonperturbed geometry is a plane wave. In other words we proved that to find the general symplectic form it is enough to find the symplectic form around the plane wave.

4.5 Plane wave symplectic form

In this subsection we evaluate the symplectic form for perturbations around the plane wave, which corresponds to putting all the barred fields in (4.10)–(4.15) to zero. At this point bulky computations are unavoidable, however they lend themselves easily to automatization. The reader may want to skip to the main results: the gravitational and the two-form symplectic current integrals (4.33),(4.38), and the symplectic form (4.39),(4.40) which agrees with (4.6). According to the arguments of the preceding subsections, this completes the proof that the symplectic form on the full moduli space is given by eq. (3.23) with the right prefactor.

Notice that once we dropped the barred fields we have $f_1 = f_5$ to the first order in a. This means that the dilaton stays constant: $\Phi = 1$, $\delta\Phi = 0$. The difference between the Einstein and string frame disappears. Also the T^4 part of the metric becomes trivial and just contributes a volume factor to the symplectic form upon integration. The unperturbed 6d metric is:

$$ds^2 = -\frac{r}{\gamma}dt^2 - dt\, dx_1 + \frac{\gamma}{r}dr^2 + r\left[\gamma d\theta^2 + 2(1-\cos\theta)(\gamma d\phi^2 - d\phi\, dy) + \frac{dy^2}{\gamma}\right] . \tag{4.25}$$

The radial part of the metric becomes regular in the variable $\rho = \sqrt{r}$. To see that the angular part is regular, one should notice that near $\theta = \pi$ the (ϕ, y) part rewrites neatly as $\gamma(2d\phi + \gamma^{-1}dy)^2$. In fact the metric of the transverse space can be brought to the flat $\mathbb{R}^4$ form, but we will find it convenient to keep working in the current coordinate system.

The pertubartions of the 6d metric after the coordinate transformation (4.21) become (the Fourier transform $\widetilde{\delta g}_{\mu\nu}$ is equal $\tilde{a}(p)$ times the coefficient given in the table):

$$\begin{aligned}
tt &: \frac{\cos\theta}{\gamma}\left[(1+r|p|)e^{-r|p|} - \chi\right] \\
tx_1 &: -\cos\theta\, r\chi\, p^2 \\
tr &: \cos\theta\left(e^{-r|p|} - \chi - r\chi'\right) ip \\
t\theta &: -\sin\theta\, r\left(e^{-r|p|} - \chi\right) ip \\
x_1 r &: \frac{\gamma\cos\theta}{r}\left(e^{-r|p|} - \chi\right) ip \\
x_1\theta &: -\gamma\sin\theta\left(e^{-r|p|} - \chi\right) ip \\
rr &: \frac{\gamma\cos\theta}{r^2}\left[(1+r|p|)e^{-r|p|} - \chi + 2r\chi'\right] \\
r\theta &: -\gamma\sin\theta\,\chi' \\
\theta\theta &: \gamma\cos\theta\left[(1+r|p|)e^{-r|p|} - \chi\right] \\
\phi\phi &: 2\gamma(1-\cos\theta)\left[(1+r|p|)e^{-r|p|} - \chi\right] \\
\phi y &: (1-\cos\theta)\left[(1+r|p|)e^{-r|p|} - \chi\right] \\
yy &: -\frac{\cos\theta}{\gamma}\left[(1+r|p|)e^{-r|p|} - \chi\right]
\end{aligned} \tag{4.26}$$

We see the effect that the presence of χ has on the regularity on the field perturbations: it smoothens the behavior at $r = 0$ by subtracting the leading singularity. A quick check of the regularity at $\theta = \pi$ for all r can be performed by noticing that the perturbation near the south pole can be written as

$$\gamma(2d\phi + \gamma^{-1}dy)^2\left[(1+r|p|)e^{-r|p|} - \chi\right]. \tag{4.27}$$

To compute the symplectic form, we choose the Cauchy surface $\Sigma = \{t = const\}$, so that the only necessary symplectic current component is J^t. The gravitational symplectic current evaluated using the Crnković-Witten formula (3.12) and the above metric perturbations is equal

$$\begin{aligned}
J_g^t(r,\theta) &= \frac{\gamma\sin\theta}{r}\int\frac{dp}{2\pi}K(p,r,\theta)\, ip\,\tilde{a}(p)\wedge\tilde{a}(-p)\,, \\
K &= e^{-2r|p|}\left[\cos^2\theta\left(3+4r|p|+2r^2p^2\right) - 1 - 2r|p|\right] \\
&\quad + e^{-r|p|}\Big\{\chi\left[\cos^2\theta\left(r^3|p|^3 - 4r|p| - 6\right) + 2 + 2r|p| - 2r^2p^2\right] \\
&\quad + \chi' r\left[\cos^2\theta\left(4+2r|p|-r^2p^2\right) + r|p| - 1\right] - \chi'' r^2\cos^2\theta\Big\}
\end{aligned}$$

$$+\cos^2\theta\,\left[\chi^2\left(3-r^2p^2\right)+\chi\chi' r\left(2r^2p^2-4\right)+\chi'^2r^2+\chi\chi'' r^2\right]$$
$$+\chi^2(r^2p^2-1)+\chi\chi' r\,. \tag{4.28}$$

We see that $K=O(r)$ for $r\to 0$ due to the presence of χ, and thus the symplectic current is regular. Integrating in θ, we get

$$\begin{aligned}\int d\theta\, J_g^t(r,\theta) &= \frac{2}{3}\gamma\int\frac{dp}{2\pi}\left[k_1+k_2+k_3\right]\, ip\,\tilde{a}(p)\wedge\tilde{a}(-p)\,,\\ k_1 &= e^{-2r|p|}(2r|p|-2)|p|\,,\\ k_2 &= e^{-r|p|}\left[\chi|p|(2-6r|p|+r^2p^2)+\chi'\left(1+5r|p|-r^2p^2\right)-\chi'' r\right]\,,\\ k_3 &= 2\chi^2rp^2+\chi\chi'\left(2p^2r^2-1\right)+r\chi'^2+r\chi\chi''\,.\end{aligned} \tag{4.29}$$

Now it remains to integrate in r. We have

$$\int dr\, k_1=-1/2\,. \tag{4.30}$$

On the other hand, a couple of integrations by part show that

$$\int dr\, k_2=-2\chi|_{r=0}=-2\,, \tag{4.31}$$
$$\int dr\, k_3=\chi^2|_{r=0}=1\,. \tag{4.32}$$

Thus the total value

$$\int dr\, d\theta\, J_G^t(r,\theta)=-\gamma\int\frac{dp}{2\pi}\, ip\,\tilde{a}(p)\wedge\tilde{a}(-p)\,. \tag{4.33}$$

Now let's consider the two form. The unperturbed value:

$$C=r\,(1-\cos\theta)\,dt\wedge d\phi+\frac{r}{\gamma}dt\wedge dy\,. \tag{4.34}$$

Again to study the unperturbed geometry one could in principle go to the regular coordinates in which the transverse metric is dX_i^2 and the three-form field strength would become $\propto dt\wedge(dX_1\wedge dX_2+dX_3\wedge dX_4)$. However, in the problem at hand such a change is unnecessary. We simply compute the two-form perturbations using (4.10)–(4.15) in (2.1) (with all the barred fields put to zero) and applying the coordinate change (4.21). They still come out singular near the south pole. An abelian gauge transformation with the paramater

$$\Lambda=\left[r\chi\cos\theta\, ip\tilde{a}(p)\right]^{\vee}dy \tag{4.35}$$

has to be applied. After that the two-form perturbations become (the Fourier transforms of the corresponding components are equal $\tilde{a}(p)$ times the coefficient in the table):

$$\begin{aligned} dt\wedge d\phi &: (1-\cos\theta)\left[(1+r|p|)\,e^{-r|p|}-\chi\right]\\ dx_1\wedge d\phi &: \gamma\sin^2\theta\, e^{-r|p|}|p|\\ dr\wedge d\phi &: \gamma(\cos^2\theta-\cos\theta)e^{-r|p|}ip\end{aligned}$$

$$
\begin{aligned}
d\theta \wedge d\phi &: \gamma \sin\theta (r|p| + 2\cos\theta) e^{-r|p|} i \,\mathrm{sign}\, p \\
dt \wedge dy &: -\frac{\cos\theta}{\gamma} \left[(1 + r|p|)\, e^{-r|p|} - \chi \right] \\
dr \wedge dy &: -\cos\theta\, e^{-r|p|} ip \\
d\theta \wedge dy &: \sin\theta\, r\, e^{-r|p|} ip
\end{aligned} \tag{4.36}
$$

This two-form is regular, and a quick way to check this is to see that near $\theta = 0$ and $\theta = \pi$ it looks like $dy \wedge (\ldots)$ and $(2d\phi + \gamma^{-1} dy) \wedge (\ldots)$, respectively.

Now we can compute the two-form symplectic current using eq. (3.13), which comes out to be

$$
J_F^t(r,\theta) = -\gamma r \sin\theta \cos^2\theta \int \frac{dp}{2\pi} e^{-2r|p|} (2r|p| + 2) i |p|^3\, \tilde{a}(p) \wedge \tilde{a}(-p)\,, \tag{4.37}
$$

$$
\int dr\, d\theta\, J_F^t(r,\theta) = -\gamma \int \frac{dp}{2\pi} ip\, \tilde{a}(p) \wedge \tilde{a}(-p)\,. \tag{4.38}
$$

We see that, unlike for the metric, in this case the χ regulator does not make its way into the symplectic current.

The total symplectic form is obtained by adding the contributions of the metric and the two-form:

$$
\Omega = \frac{1}{(2\pi)^7 g_s^2} (2\pi)(2\pi R)(2\pi)^4 V_4 \int dr\, d\theta\, (J_G^t + J_F^t)\,, \tag{4.39}
$$

where we took into account volume factors appearing because of the integration over the ϕ and the y circles, and over T^4. All in all, the final result is

$$
\Omega = -\frac{1}{2\pi\mu^2} \int \frac{dp}{2\pi} ip\, \tilde{a}(p) \wedge \tilde{a}(-p)\,, \tag{4.40}
$$

which is equivalent to (4.6). Q.E.D.

5. Discussion

In this paper we have quantized the moduli space of the regular D1-D5 geometries, and used this result to count the D1-D5 black hole microstates, directly from SUGRA. The method is based on our previous work [2, 3], and incorporates a new ingredient — the consistency condition — which allowed us to reduce the computational work needed to obtain the general result. Most certainly, this new ingredient will play a role in the future applications of the symplectic form quantization to the other SUGRA moduli spaces, such as the 3-charge moduli space parametrized by hyper-Kaehler manifolds [9].

Our result fits nicely with the general idea of Mathur's program [1], which can be loosely described as aiming to understand the inner structure of quantum black holes purely from SUGRA, i.e. without D-branes. Recently, several interesting results breathed new life into the program and opened up new avenues of research. In [13], it was demonstrated how the emergence of an effective black hole geometry may be seen from AdS/CFT correlators in a typical D1-D5 ground state. In [14], it was shown how an effective geometry may be recovered by studying boundary 1-point functions. It would be very interesting to combine these two approaches and derive a truly emergent horizon of the D1-D5 black hole. Another problem which needs to eventually be addressed is the derivation of the nonzero area horizon from the curvature-corrected SUGRA Lagrangian. In the best of the worlds, these two derivations would produced horizons of the same size. The future will show if this is indeed the case.

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